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Reliability of Environmental Systems

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Major reliability indices used in environmental engineering

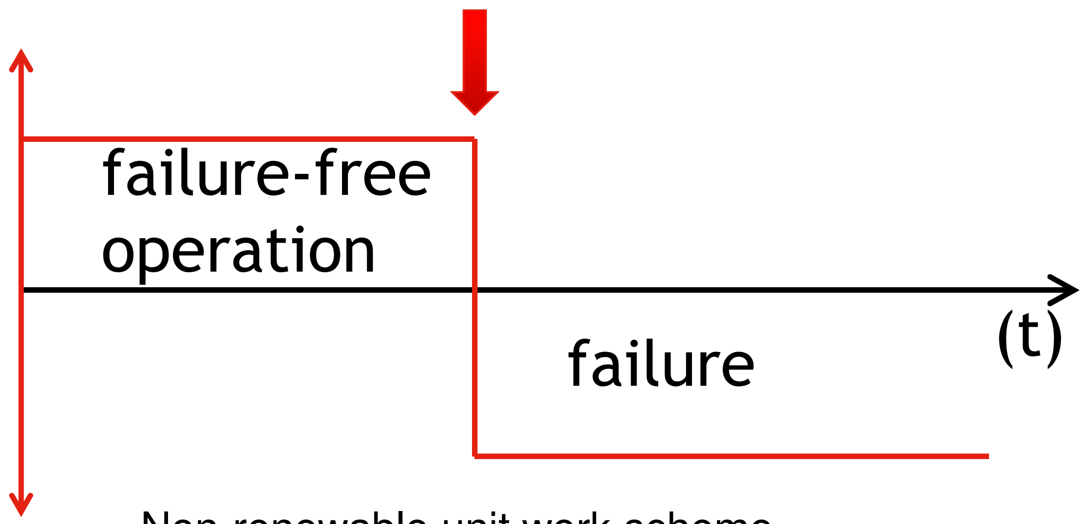
- For quantitative reliability assessment of systems **reliability indices** were introduced
- Reliability indices can be expressed in form of **functional** or **numerical** characteristics which include properties of the system and its elements as well as random processes connected with functioning of the system

Renewable and non-renewable units

- Smallest piece of the system (indivisible) is an **units (elements)**
- We can define **non-renewable** and **renewable** units
- We will discuss these two types of units separately

Non-renewable units

- **Non-renewable units** cannot be repaired (or the repair is not cost-effective)



Non-renewable unit work scheme

Non-renewable units – probability of failure-free operation

- Practical measure for assessment of non-renewable units operation is a **probability (R (t)) that the unit will be failure-free** at the beginning of operation (t = 0) and specific period of time (0, t]:

$$R(t) = P(T \square t)$$

T – continuous random variable describing the operation time (up to the moment of failure)

Non-renewable units

- **Probability of failure (U(t))** is opposite to probability of failure-free operation (R(t)):

$$U(t) = 1 - R(t) = P(T < t)$$

- For time range from t ≥ 0 it is also distribution (F(t)) of a random variable T describing failure-free operation time of the unit:

$$F(t) = U(t) = P(T < t)$$

Non-renewable units (contd)

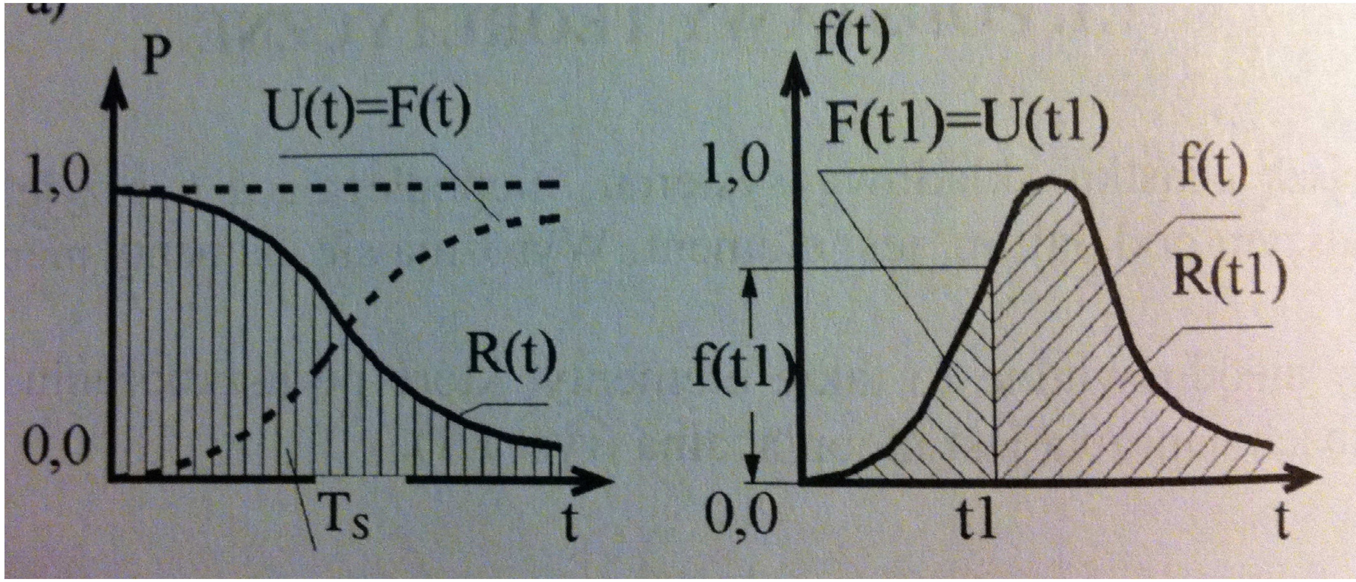
If functions R(t) and F(t) are continuous we can express them in form of **distribution density function f(τ)**:

$$F(t)=\int_0^t f(\tau)\,d\tau \quad \text{and} \quad R(t)=\int_t^{+\infty} f(\tau)\,d\tau$$

Transforming further, we obtain the **probability density of random variable T**:

$$f(t)=\frac{dF(t)}{dt}=-\frac{dR(t)}{dt}$$

Non-renewable units – Graphs of R(t), F(t) and f(t) functions



Failure intensity of non-renewable units

Failure intensity (rate) of non-renewable units:

$$\lambda(t) = \frac{f(t)}{R(t)}$$

after transformation, we obtain:

$$\lambda(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} = -\frac{d}{dt}\left[\ln R(t)\right]$$

this leads to the **general Wiener equation** – the **general equation for reliability**:

$$R(t) = \exp - \int_0^t \lambda(\tau)\, d\tau$$

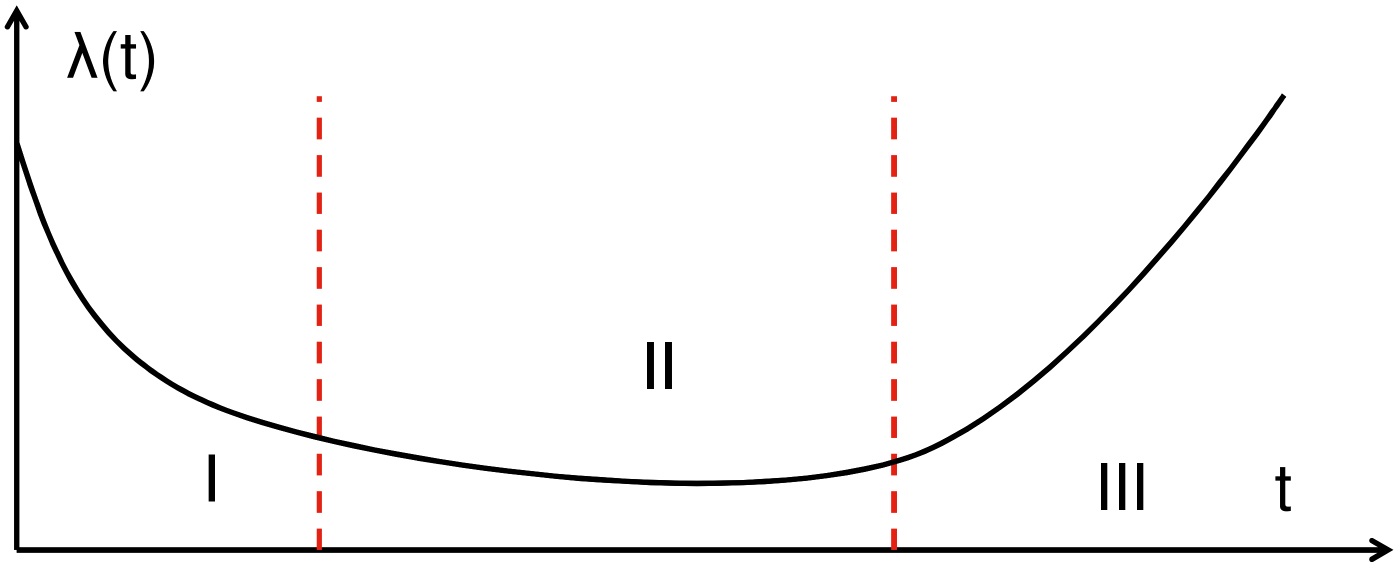
for any continuous failure intensity λ(t)

Non-renewable units – numerical characteristics of reliability

- Apart from **functional characteristics** (R(t), F(t), f(t) and λ(t)) we can use **numerical characteristics**
- For non-renewable units we use **mean time to failure (MTTF)** - $T_u = E(T)$
- **Mean time to failure** is a expected value (**E**) of random variable **T**: $\mathbf{T_u = E(T)}$ and (see R(t) function chart above):

$$T_u = \int_0^{\infty} R(\tau)\, d\tau$$

Typical run of a failure intensity function λ(t) for a technical object (Bathtub curve)



I – period of running-in, high failure intensity but quickly **decreasing** (elimination manufacturing defects)
II – period of normal (regular) operation of the system – we can assume failure intensity to be **constant**
III – period of system ageing, failure intensity is **increasing** rapidly

Operation period

In most analyses only **period II** – the normal operation time is considered – for this period failure intensity is **constant** (when considered in longer operation time) then:

$$\lambda(t) = \lambda = const$$

For such assumption, random variable describing the time of failure-free operation period (T) has an **exponential** character (distribution)

Operation period (cont.)

- The general equations can be simplified to the following forms:

$$R(t) = e^{-\lambda t} = \exp - \lambda \cdot t$$

$$U(t) = F(t) = 1 - e^{-\lambda t} = 1 - \exp - \lambda \cdot t$$

$$f(t) = \lambda \cdot e^{-\lambda t} = 1 - \exp - \lambda \cdot t$$

$$T_u = \frac{1}{\lambda}$$

For **exponential distribution** failure intensity – λ can be interpreted as a **mean number of failures per unit of time**

"No memory condition"

- For **exponential** distribution the "**no memory condition**" is fulfilled that can be put down using *conditional probability*:

$$R(t+\Delta t|t) = R(\Delta t)$$

When we transform above with: $R(t) = e^{-\lambda t} = \exp - \lambda \cdot t$ we obtain:

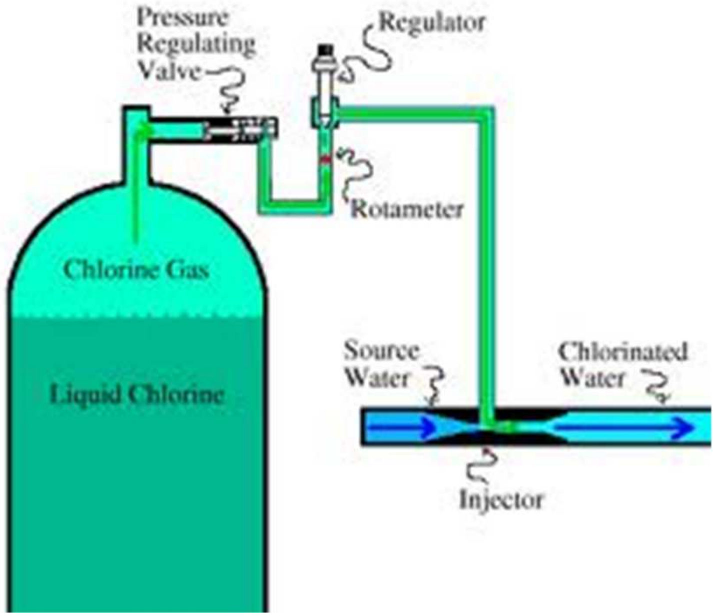
$$R(t+\Delta t|t) = \frac{R(t+\Delta t)}{R(t)} = \frac{\exp - \lambda (t+\Delta t)}{\exp - \lambda \cdot t} = \exp - \lambda \cdot \Delta t = R(\Delta t)$$

"No memory condition"

- In the context of the last equation "*no memory*" means that the probability of the unit failure in the period of time (Δt) is dependent **only** on the duration of the period and **not on how much time this unit worked BEFORE**
- Unit which time of failure-free operation can be described by exponential distribution in **every instant of its operation can be treated "as new"**
- The **exponential distribution** and **"no memory condition"** assumptions and characteristics can be applied to both **non-renewable** as well as **renewable units**

Example 1

Failure intensity of the (non-renewable) chlorinator is equal $\lambda = 1e^{-6}$ 1/h. Calculate the probability of chlorinator failure-free operation during time period of $t = 1e^{5}$ hours. Assume exponential distribution.



Answer 1

- The failure-free operation probability is expressed by the function in form of:

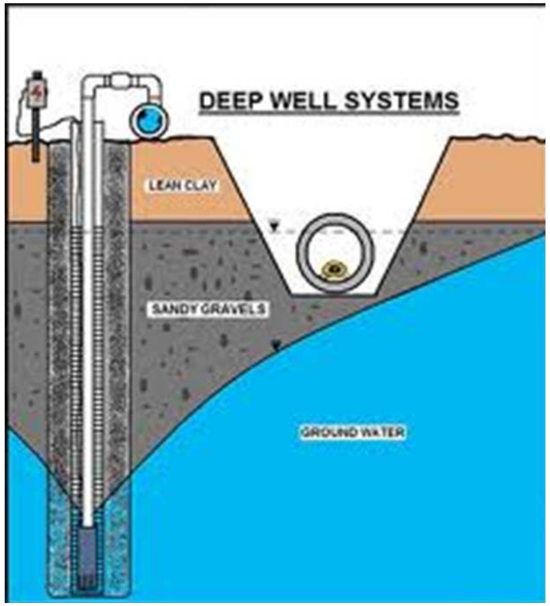
$$R(t) = \exp - \lambda \cdot t$$

For data given in the example we obtain reliability:

$$R(t) = \exp - 1e^{-6}\, 1/h \cdot 1e^{4}\, h = \exp - 0.01 \cong 0.99$$

Example 2

- In deep well electronic controller for "dry-run" protection of borehole pump was installed. Manufacturer determined its failure-free operation time to 5 years and set 2 year warranty. Assuming that controller is non-renewable unit, calculate probability of failure-free operation during the 2 years of warranty



Answer 2

- Assuming that distribution of controllers failures is exponential and calculating value failure intensity as a reciprocal of mean uptime T_u

$$\lambda = \frac{1}{T_u} = \frac{1}{5 \text{ years}} = 0.20\, 1/\mathit{year}$$

$$R(t = 2 \text{ years}) = \exp - 0.20 \cdot 2\, \mathit{years} = \exp - 0.4 \cong 0.67$$

Example 3

- Calculate probability that non-renewable unit will be failure-free during time period of $(0, T_u]$.

Answer 3

Probability that unit will be failure-free until t can be expressed by equation:

$$R(t) = e^{-\lambda t}$$

hence for $t = T_u$ we can write:

$$R(T_u) = e^{-\lambda T_u} = e^{-1} = 0.367879$$

In practice it means that approx. 37% of non-renewable units life will be longer then mean failure-free time T_u while 63% of them will fail before.

Question 4

- Calculate probability of failure-free operation for $R(t_2)$ = 10 000 hours and mean failure-free time T_u for non-renewable unit having:
  - t_1 = 5 000
  - $R(t_1)$ = 0.9875

Answer 4

We can start with finding failure intensity using known relation for exponential distribution:

$$e^{-\lambda t_1} = 0.9875$$

we take a logarithm for both sides of the equation:

$$-\lambda \cdot t_1 = \ln 0.9875$$

$$-\lambda \cdot t_1 = -0.012578$$

$$\lambda = \frac{0.012578}{t_1} = \frac{0.012578}{5000} = 2.5e^{-6}\ \ 1/\mathrm{h}$$

Answer 4 (cont.)

For known λ = 2.5e^{-6} we can now calculate reliability of the unit:

$$R(t_2) = e^{-\lambda t_2} = e^{-2.5e-6 \cdot 10000} = 0.975310$$

The MTTF is then:

$$T_u = \frac{1}{\lambda} = \frac{1}{2.5e^{-6}} = 400\,000 \text{ hours}$$

Exercise 5

- From large number of non-renewable units ($n = 5000$) after time $t = 1000$ hours 75 units failed. Calculate MTTF

Answer 5

For a time **t** empirical reliability of the units is equal to:

$$R(t) = 1425/1500 = 0.95$$

and can be expressed in exponential form:

$$R(t) = e^{-\lambda t}$$

After some modifications we obtain failure intensity:

$$\lambda = \frac{\ln R(t)}{t} = \frac{\ln 0.95}{1000} = 5.13e^{-5} \ \ 1/\text{h}$$

thus MTTF is equal:

$$T_u = \frac{1}{\lambda} = \frac{1}{5.13e^{-5}} \cong 19\,500\,\text{hours}$$

Exercise 6

MTTF of non-renewable units is equal T_u = 4000 hours. Calculate number (share) of elements which will fail in consecutive periods of time Δt = 2000 hours.

Answer 6

We can reformulate this assignment to find the decrease in reliability for following periods Δt of time.
We start with finding the failure intensity which is equal:

$$\lambda = \frac{1}{T_u} = \frac{1}{4000} = 2.5e^{-4} \ \ 1/\mathrm{h}$$

The drop in reliability for the first period of time Δt will be:

$$R(I) = R(t=0) - R(t=\Delta t) = e^{0} - e^{-\lambda \Delta t} = 1 - 0.606531 = 0.393469$$

so 39.3 % of elements will fail after 2000 hours

Answer 6 (cont.)

Drop in reliability in second period [Δt, 2Δt] will be:

$$R(II) = R(t = \Delta t) - R(t = 2\Delta t) = e^{-\lambda\Delta t} - e^{-\lambda 2\Delta t} = 0.606531 - 0.367879 = 0.238652$$

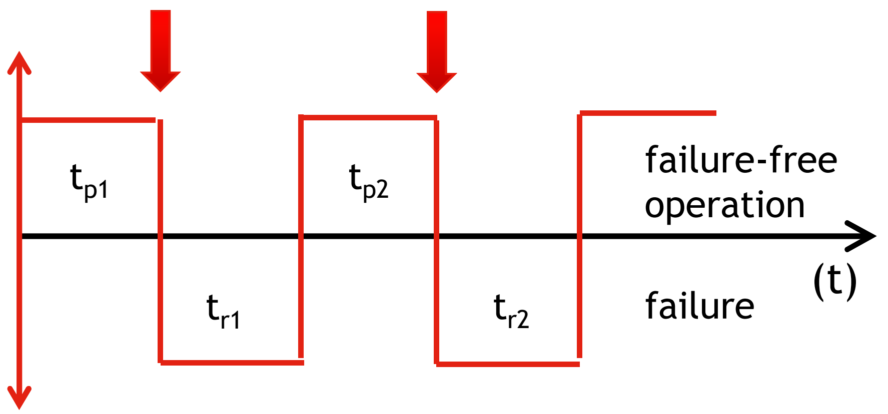
and third:

$$R(III) = R(t = 2\Delta t) - R(t = 3\Delta t) = e^{-\lambda 2\Delta t} - e^{-\lambda 3\Delta t} = 0.367879 - 0.223130 = 0.144749$$

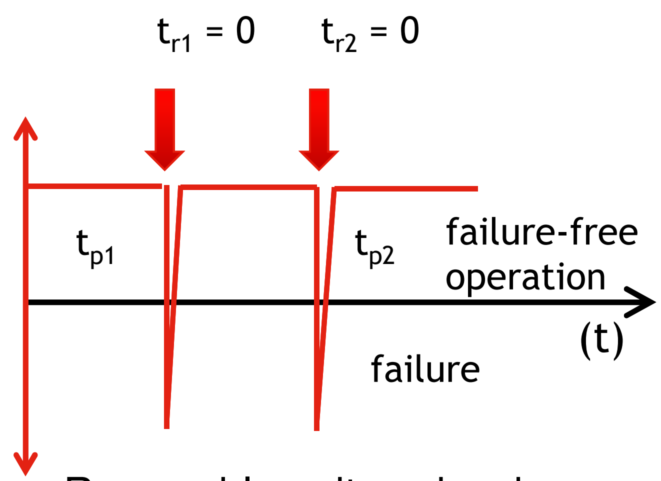
Summing up: after three consecutive periods of 2000 hours 39.4%, 23.9% and 14.5% of units will fail.

Renewable units

- Renewable units can be **repaired** (or renewed) after they fail
- After renewable units is repaired it is included in the system and treated "as new"
- In longer period of operation time there maybe multiple cycles of failure and failure-free modes
- There are two types of renewal:
  - **instant renewal** (the T_r is negligible)
  - **real renewal time** ($T_r > 0$)



Renewable unit work scheme – **real renewal time**



Renewable unit work scheme – **instant renewal time**

Renewable units

- Operation of renewable units can be assessed by analysis of **time between failures** (e.g. MTBF) or analysis of **number of events** in given time period - **event** is either the **occurrence of failure** or the **end of repair**
- Assumptions:
  - repair of an unit is complete ("as new")
  - units can be replaced with new one
  - units are repaired just after they breakdown (time to repair = 0)
  - distribution of both time between failures and time of failure is exponential
  - only so called normal operation period is considered (period II)

Time between failures analyses

- When the time between failures is considered in analyses we can distinguish two random variables $\mathbf{T'_{up}}$ – time when unit is operating and $\mathbf{T'_{down}}$ – time when unit is damaged and not operational
- The expected values (**E**) of this variables are **mean time between failures** (MTBF - T_{up}) and **mean time of failure (**T_{down}**):**

$$E(T'_{up}) = T_{up} \quad \text{and} \quad E(T'_{down}) = T_{down}$$

Mean working (up time) - T_u

- **Mean time between failures (T_{up})** – time period of up time between failures. It is an expected value (E) of random variable T_u which describes the uptime of the system (or its elements) between two consecutive failures:

$$T_{up} = E(T_u) = \int_0^{\infty} t \, f(t)\, dt$$

Applied for operating (real) data yields:

$$T_{up} = \frac{1}{k+z}\left(\sum_{i=1}^{k} t_{i-up} + z \cdot t \right), d$$

E(T_u)
t - ob
t_{i-up} -
k - nu
z - nu
f(t) -

Mean time to repair (recovery) - T_{down}

- **Mean time to repair** is a expected value of random variable T_r which describes repair time:

$$T_{down} = E\left(T_{down}\right) = \int_0^{\infty} t \; f_n(t)\, dt$$

Applied for operating (real) data yields:

$$T_{down} = \frac{1}{n_o}\sum_{i=1}^{n_o} t_{i-down}, \; d$$

n_o - number of repairs in examined period

t_{i-down} - duration of i-repair

Number of events analyses

- When we observe number of events in given time period we are analyzing series of for events (start of down time and end of repair) so called **streams of failures (ω)** and **streams of repairs (μ)**
- Assuming exponential distribution of random variable T_{up} and T_{down} the parameters ω and μ are also constant and the stream of failures (ω) can be substitute by **parameter λ** and it is now called the **failure intensity (rate)**:

$$\omega(t) = \omega = \lambda$$

Failure rate of renewable units

- Failure rate λ is describing the mean number of downtime per unit time:

$$E(\mathbf{L}) = \lambda$$

where **L** is a random variable describing the number of downtime per unit time

- For exponential distribution we can write by analogy:

$$T_{up} = \frac{1}{\lambda} = MTBF \qquad T_{down} = \frac{1}{\mu}$$

- failure rate λ is at the same time parameter at discrete Poisson distribution which is by expression E(L) = λ describing number of failures per unit time

Failure intensity - λ

- Failure intensity can be obtained from eq.:

$$\lambda(t) = \frac{dE(T_{up})}{dt}$$

Applied for operating (real) data yields:

$$\lambda(t) = \frac{1}{T_{up}}, 1/d$$

$$\lambda(t) = \frac{n(t, t+\Delta t)}{N \cdot \Delta t}$$

n(t,t+Δt) – number of all failures in examined period
N – number of examined objects (or for linear objects = length)
Δt – observation period

Repair intensity - μ

- **Repair (recovery intensity) - μ** is defining the number of failures repaired in a time unit:

$$\mu(t) = \frac{n(t, t + \Delta t)}{n(t) \cdot \Delta t}$$

$$\mu = \frac{1}{T_{down}}, 1/d$$

n(t,t+Δt) – number of objects which recovery ended in time interval (t, t+Δt)
n(t) – number of objects, which recovery ended at t
Δt – observation period

Renewable units

- For renewable units T_{up} and T_{down} are single measure of reliability
- We can use **complex measure** which incorporates all single measures
- In practice we use:
  - Standby index (stationary and non-stationary)
  - Standstill index
  - Operational standby index

Standby index - K_s

- **(Non-stationary) Standby index $K_s(t)$** describes a probability that system will be in operation at a given time (t). With increasing operating time, value of standby index approaches the boundary value - **stationary standby index K_s**

$$K_g(t) = \frac{\mu + \lambda \exp\left[-(\mu+\lambda)\cdot t\right]}{\mu+\lambda}$$

$$K_g = \lim_{t\to\infty} K(t) = \frac{\mu}{\mu+\lambda} = \frac{T_{up}}{T_{up}+T_{down}} = \frac{T_{up}/T_{down}}{T_{up}/T_{down}+1}$$

Standby index - K_s

- Standby index is a major complex measure for renewable units
- Stationary standby index is interpreted a probability that in any moment (but long enough) from the beginning of operation, unit will be working – commonly we called it **reliability**

Standstill (parking) index - K_p

- Standstill or parking index (K_p) is complementing the standby index (K_s):

$$K_p = 1 - K_s = \frac{T_{down}}{T_{up} + T_{down}}$$

Operational standby index - K_o

- **Operational standby index** is another complex measure used for describing reliability of the systems:

$$K_o(\Delta t) = K_s R(\Delta t)$$

- **Operational standby index** is interpreted as a probability that in any moment (but long enough) from the beginning of operation, unit **will be fully operable and that it will sustain this state for a time period of Δt**

Binomial distribution

- Number of failures of renewable units can be described by **discrete dimetric Bernoulli distribution** or discrete Poisson distribution. Binomial distribution in the form of:

$$P(k,n,p)=\binom{n}{k}p^{k}(1-p)^{n-k}$$

describes the probability of obtaining k – successes in n – independent trials, whereby the probability of success in single trial is p

Binomial distribution – interpratation in reliability

- For independent operation of n – units and assuming probability p of single unit, the equation for binomial distribution allow for calculation of probability of simultaneous occurrence of k = 0, 1, 2,…, n failures
- Binomial or Bernoulli distribution is used for **small number of n (failures)**

Poisson distribution

- Poisson distribution is a limiting case of binomial distribution
- For large number of independent trials (n -> ∞), probability of a success is small (p -> ∞) and product of n and p is constant n x p = λ (failure intensity). The limiting probability of occurrence of *k* failures is given by the equation:

$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Poisson distribution

- In practice equation is used when $p < 0.1$ and $n > 50$
- It may be used for $n > 10$ but the final result may deviate from result obtained from binomial distribution
- In case of considering N simultaneously working uniform units (assuming instant renewal) instead of λ we use failure rate of group of units: $\Lambda = N \times \lambda$
- When operation of a single unit at any time **t** is considered the failure intensity in the following form must be used $\Lambda = \lambda \times t$

Exercise 1

- An unit is about to be put to operation. Unit is characterized by MTBF = 5000 h and T_{down} = 16 hours. Determine time needed for standby index K_s to reach constant value.

Answer 1

We must calculate time when value of non-stationary standby index $K_s(t)$ differ by small value $\varepsilon > 0$ from stationary standby index K_s.
We can find K_s from:

$$K_s = \frac{T_{up}}{T_{up} + T_{down}} = 0.996810$$

Let's assume error $\varepsilon = 0.0005$, so we need to find time t when $K_s(t) = K_s + \varepsilon = 0.99731$. Transforming equation for non-stationary standby index:

$$K_s(t) = \frac{\mu + \lambda \cdot \exp-(\mu+\lambda)t}{\mu+\lambda} = 0.996810$$

$$\exp-(\lambda+\mu)t = \frac{1}{\lambda}\left[\left(K_s+\varepsilon\right)\left(\mu+\lambda\right)-\mu\right]$$

Answer 1 cont.

After taking a logarithm and transform:

$$t=-\frac{1}{\lambda+\mu}\ln\frac{1}{\lambda}\left[(K_s+\varepsilon)(\mu+\lambda)-\mu\right]$$

Failure and repair intensity can be calculated from:

$$\lambda=\frac{1}{T_{up}}=\frac{1}{5000}=2e^{-4}\,1/h$$

$$\mu=\frac{1}{T_{down}}=\frac{1}{16}=6.25e^{-2}\,1/h$$

Then finally time:

$$t=-\frac{1}{2e^{-4}+6.25e^{-2}}\ln\frac{1}{2e^{-4}}\left[(0.99681+0.0005)\left(2e^{-4}+6.25e^{-2}\right)-6.25e^{-2}\right]=29.56\,h$$

Exercise 2

Assess which of two segments of pipeline have higher reliability:

- A – pipe made of steel of length L(A) = 10 km
- B – pipe made of cast iron L(B) = 2 km

Values of unit failure intensities for these pipelines are:

- $\lambda_o(A) = 0.005$ 1/km a
- $\lambda_o(B) = 0.05$ 1/km a

Pipelines are laid in different ground conditions and have different mean time to failure $T_{down}(A)$ = 15 hours and $T_{down}(B)$ = 10 hours.

Answer 2

For linear units failure intensity can be obtain from:

$$\lambda = \lambda_o L$$

Failure intensities for pipelines A and B are:

$$\lambda(A) = 0.005 \cdot 10 = 0.05\, 1/annum$$

$$\lambda(B) = 0.05 \cdot 2 = 0.10\, 1/annum$$

MTBFs are:

$$T_{up}(A) = \frac{1}{\lambda} = \frac{1}{0.05} = 20\ years = 175\,200\ hours$$

$$T_{up}(B) = \frac{1}{\lambda} = \frac{1}{0.1} = 10\ years = 87\,600\ hours$$

Answer 2 cont.

Reliability of pipelines A and B are:

$$K_s = \frac{T_{up}}{T_{up} + T_{down}}$$

$$K_s(A) = 0.999914$$

$$K_s(B) = 0.999886$$

Exercise 3

A water treatment plant (as an undividable unit) can treat water with **typical** train of processes for normal quality of raw water. In case of unusual fluctuations of quality WTP can treat water using **ancillary** train of processes. MTBF and MTTF of the ancillary train: T_{up} = 8000 hours and T_{down} = 12 hours.
Find probability that WTP will be able to treat water during extremely low water quality for Δt = 2 days.

Answer 3

To solve the problem we need to find the value of operational standby index K_o.

$$K_o(\Delta t) = K_s R(\Delta t)$$

Because the occurrence of extreme conditions (poor water quality) is random, so probability that alternative train will be ready in any moment can be found from equation:

$$K_s = \frac{T_{up}}{T_{up} + T_{down}} = 0.998502$$

Whereas the probability of continuous, failure-free operation during time Δt is expressed by equation: $R(t) = e^{-\lambda t}$

Failure intensity: $\lambda = 1/T_{up}$

Answer 3 cont.

The probability of failure-free operation during time period of Δt is equal:

$$R(\Delta t = 2\ days) = 0.994018$$

Hence the probability that WTP will be able to operate during Δt:

$$K_o(\Delta t) = K \cdot R(\Delta t) = 0.992529$$

Exercise 4

Ground water intake is composed of 5 independent deep wells. Reliability of each deep well is described by stationary standby index $K_s = 0.98$. What is the probability that in any moment 4 deep wells will be operable at the same time.

Answer 4

We are looking for probability that in any moment number of non-working deep wells will be equal to 0 or 1. Assuming to Bernoulli equation the probability of success (in this case – failure) referring to any moment of time p = 1 – K_s = 0.02 we obtain:

$$P(k=0) = P(k=0, n=5, p) = \binom{5}{0} p^0 (1-p)^5 = K_s^5 = 0.903921$$

$$P(k=1) = P(k=1, n=5, p) = \binom{5}{1} p^1 (1-p)^4 = 5(1-K_s)K_s^4 = 0.092237$$

Answer 4 cont.

The overall probability is equal:

$$P(k \le 1) = P(k = 0) + P(k = 1) = 0.996158$$

Exercise 5

In 10 story block of there are 4 flats on each floor. Find probability that during t = 1 year, every single valve (one per apartment) will be operable. Assume failure intensity of the valve $\lambda(\text{valve}) = 0.08\ \text{year}^{-1}$

Answer 5

The probability of failure-free operation of the valve during time t is equal:

$$R(t) = e^{-\lambda(valve)t}$$

Thus the probability of valve failure in this period is equal:

$$F(t) = 1 - R(t) = p = 0.076884$$

Answer 5 cont.

Total number of valves for a block is n = 10 x 4 = 40 pieces. For this case we will use Poisson distribution. The failure rate of valves in all apartments is equal:

$$\Lambda = n \cdot \lambda(valve) = 3.2 \ year^{-1}$$

and probability of failure-free run of all valves (or occurrence of k = 0):

$$P(k=0) = \frac{\Lambda^k}{k!} e^{-\Lambda} = e^{-\Lambda} = 0.040762$$

Exercise 6

Ground water intake is composed of five dug wells. The MTBF = 8 years (T_{up}) and MTTF = 2 days (T_{down}) were determined upon operational data. To achieve full capacity intake must work with three active wells. Find reliability of intake defined by probability that it will sustain maximum capacity at any time.

Answer 6

For small number of uniform elements (n = 5) we can use binomial distribution. For number of successes (in this case well failure) k = 0, 1 and 2 (thus there will be still 3 wells left working) and probability p of success (failure of one well) at any time (where $p = K_p$ – standstill index):

$$P(k,n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

Answer 6 cont.

We start with finding standstill index K_p:

$$K_p = \frac{T_{down}}{T_{up} + T_{down}} = \frac{2\,days}{2920\,days + 2\,d} \cong 0.00068$$

Next we can find probability of occurrence of each operating case: failure-free run of all (k = 0), failure of one (k = 1) and two wells (k = 2)

Answer 6 cont.

$$P(k=0) = P(k=0, n=5, p=0.00068) = \binom{5}{0} p^0 (1-p)^5 =$$

$$= (1-0.00068)^5 \cong 0.006604621$$

$$P(k=1) = P(k=1, n=5, p=0.00068) = \binom{5}{1} p^1 (1-p)^4 =$$

$$= 5 \cdot 0.00068 (1-0.00068)^4 = 0.003390761$$

$$P(k=2) = P(k=2, n=5, p=0.00068) = \binom{5}{2} p^2 (1-p)^3 =$$

$$= 10 \cdot 0.00068^2 (1-0.00068)^3 = 0.000004614$$

Answer 6 cont.

The overall probability will include all analyzed cases, when capacity will be above required:

$$P(k \square 2) = P(k = 0) + P(k = 1) + P(k = 3) = 0.99999996$$

Reliability schemes

Exercise 1

In WTP there are 6 (same) rapid filters. The maximum capacity of the filter is equal Q(F) = 20% Q_n (Q_n – nominal capacity of the WTP). What is reliability scheme of the filters?

Answer 1

The ratio $n = Q_n/Q(F) = 5$ is a number of active (indispensible) filters. Required capacity will be sustained by 5 active – so the structure is 5 from 6.

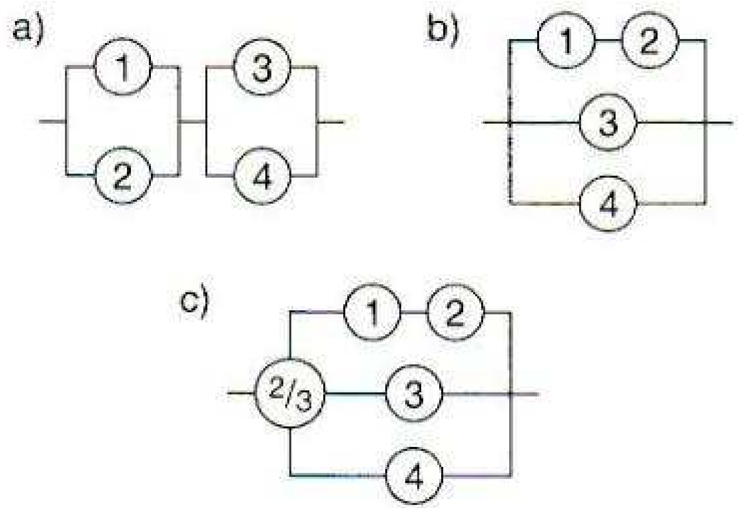
Exercise 2

In pumping station there are 4 pumps with capacities equal: $Q1 = Q2 = 1/3Q_n$ (nominal capacity of the pumping station), $Q3 = Q4 = 2/3Q_n$. The pumps are connected in parallel (pipes and fixtures can be omitted in this example). Find reliability schemes for the required capacity (Q_r) equal:

a) $Q_r = Q_n$
b) $Q_r = 2/3Q_n$
c) $Q_r = Q_n + 1/3Q_n$ (for fire fighting)

Answer 2

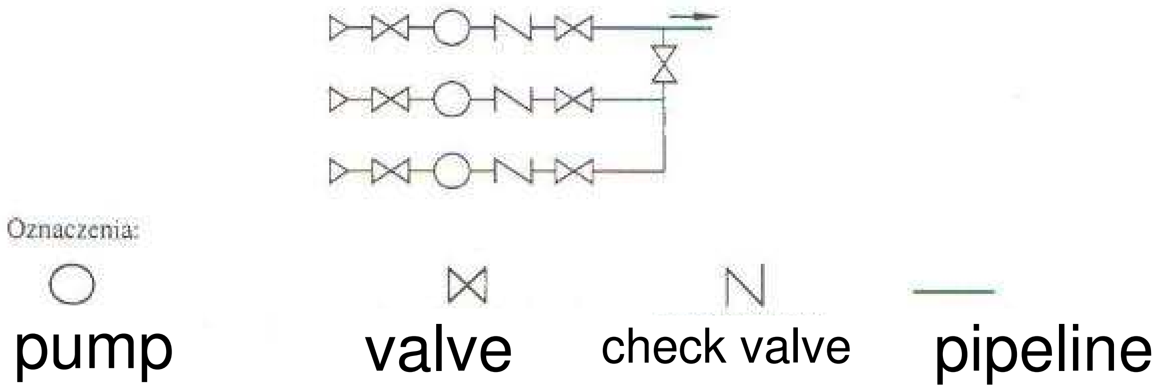
Reliability scheme depends on the actual water demand. In first case (a) the nominal capacity can be obtained by activating one $1/3Q_n$ pump and one $2/3Q_n$ pump:



Exercise 3

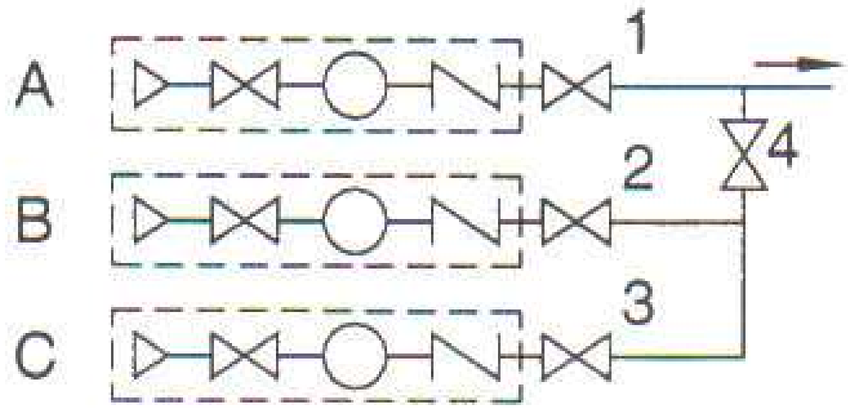
For pumping station (technical schemes below) find reliability schemes for three different cases (pipelines can be omitted):

a) 1 from 3
b) 2 from 3
c) 3 from 3



Answer 3

To obtain clear schemes and to facilitate analyses, pumping station elements are grouped into blocks (A, B, C).
Only a fatal failure of valve (V) and check valve (CV) will be considered. Then elements in serial and connected with other paths (1, 2, 3) will not be blocked



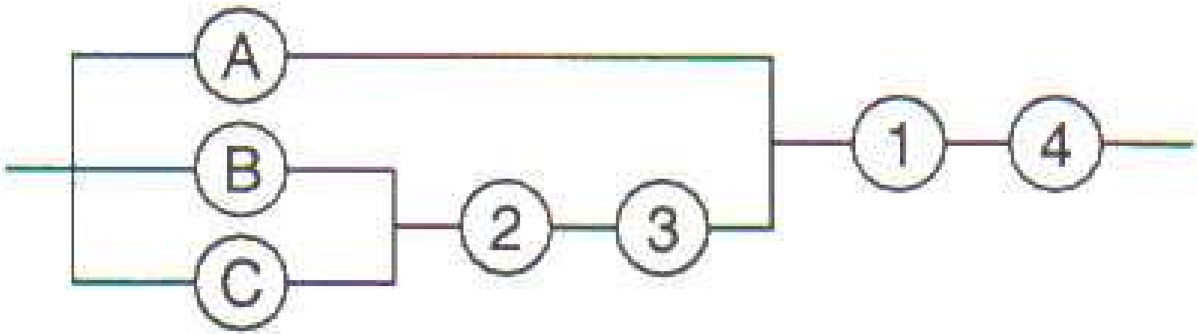
Answer 3 cont.

We will list failure-free paths (of flow). Case 1 from 3 means that there must be one pump working. Thus number of failure-free paths are equal $\binom{3}{1} = 3$ they are D1, D2 and D3. The elements and blocks are presented in the table:

Path	Blocks and elements						
D1	A			1			4
D2		B		1	2	3	4
D3			C	1	2	3	4

Answer 3 cont. (1 from 3)

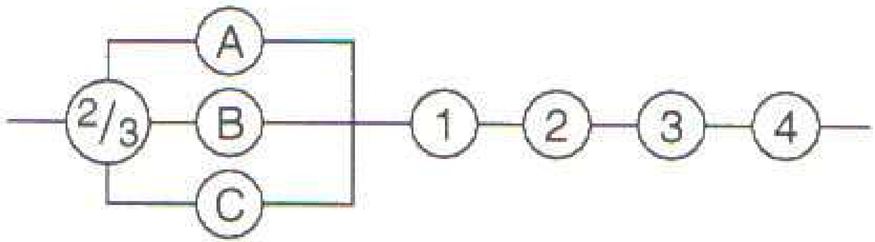
For “1 from 3” case, pumping station will be working when one of the blocks A, B and C will be working. Elements no 1 and 4 are present in all paths (so they must be put in serial). Elements 2 and 3 are on two paths D2 and D3. The reliability scheme “1 from 3”



Answer 3 cont. (2 from 3)

For "2 from 3" case, number of paths $\binom{3}{2} = 3$ D4, D5 and D6.

Droga	Bloki i elementy						
D4	A	B		1	2	3	4
D5	A		C	1	2	3	4
D6		B	C	1	2	3	4



Answer 3 cont. (3 from 3)

Case 3 from 3 – only one path D7 is possible because all pumps must be working